Lösungen

AB Aufgabe 4

(a) $f(x) = -x^2 + 4; x_0 = 2$

$$\lim_{h\to 0} D(h) = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h} = \lim_{h\to 0} \frac{-(2+h)^2 + 4 - (-2^2+4)}{h} = \lim_{h\to 0} \frac{-4h+h^2}{h} = \underline{-4}$$

(b) $f(x) = 3x + 1; x_0 = -2$

$$\lim_{h\to 0} D(h) = \lim_{h\to 0} \frac{3(-2+h) + 1 - (3\cdot(-2)+1)}{h} = \lim_{h\to 0} \frac{3h}{h} = \underline{3}$$

(c) $f(x) = x^3; x_0 = 2$

$$\lim_{h\to 0} D(h) = \lim_{h\to 0} \frac{(2+h)^3 - (2^3)}{h} = \lim_{h\to 0} \frac{2^3 + 12h + 6h^2 + h^3 - 2^3}{h} = \lim_{h\to 0} 12 + 6h + h^2 = \underline{12}$$

(d) $f(x) = x^2 - x; x_0 = 1$

$$\lim_{h\to 0} D(h) = \lim_{h\to 0} \frac{(1+h)^2 - (1+h) - (1^2-1)}{h} = \lim_{h\to 0} \frac{1 + 2h + h^2 - 1 - h - 0}{h}$$
$$= \lim_{h\to 0} \frac{h+h^2}{h} = \underline{1}$$

(e) $f(x) = \frac{12}{x+2}; x_0 = -1$

$$\lim_{h\to 0} D(h) = \lim_{h\to 0} \frac{\frac{12}{-1+h+2} - \frac{12}{-1+2}}{h} = \lim_{h\to 0} \frac{\frac{12}{h+1} - 12}{h} = \lim_{h\to 0} \frac{12}{(h+1)h} - \frac{12}{h}$$
$$= \lim_{h\to 0} \frac{12 - 12(h+1)}{(h+1)h} = \lim_{h\to 0} \frac{-12h}{(h+1)h} = \lim_{h\to 0} \frac{-12}{(h+1)} = \underline{-12}$$

(f) $f(x) = 3x^2 + a; x_0 = 2$

$$\lim_{h\to 0} D(h) = \lim_{h\to 0} \frac{3(2+h)^2 + a - (3\cdot 2^2 + a)}{h} = \lim_{h\to 0} \frac{12 + 12h + 3h^2 + a - 12 - a}{h}$$
$$= \lim_{h\to 0} \frac{12h + 3h^2}{h} = \lim_{h\to 0} 12 + 3h = \underline{12}$$

(g) $f(x) = x^2; x_0$ beliebig

$$\lim_{h\to 0} D(h) = \lim_{h\to 0} \frac{(x_0+h)^2 - x_0^2}{h} = \lim_{h\to 0} \frac{x_0^2 + 2x_0h + h^2 - x_0}{h} = \lim_{h\to 0} \frac{2x_0h + h^2}{h}$$
$$= \lim_{h\to 0} 2x_0 + h = \underline{2x_0}$$

(h) $f(x) = |x|; x_0 = 0$

$$\lim_{h\to 0} D(h) = \lim_{h\to 0} \frac{|0+h| - |0|}{h} = \lim_{h\to 0} \frac{|h|}{h} \text{ existiert nicht, da}$$
$$\lim_{h\to 0+} \frac{|h|}{h} = 1 \text{ und } \lim_{h\to 0-} \frac{|h|}{h} = -1$$

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(b)

$$f'(3) = \lim_{h\to 0} \frac{-2(3+h)^2 - (-2\cdot 3^2)}{h} = \lim_{h\to 0} \frac{-2(9+6h+h^2)+18}{h} = \lim_{h\to 0} \frac{-12h-2h^2}{h} = \underline{-12}$$

(d)

$$f'(1) = \lim_{h\to 0} \frac{2(1+h)^2 - 2\cdot 1^2}{h} = \lim_{h\to 0} \frac{4h+2h^2}{h} = \underline{4}$$

(e)

$$f'(-1) = \lim_{h\to 0} \frac{\frac{1}{-1+h} - \frac{1}{-1}}{h} = \lim_{h\to 0} \frac{\frac{1+(h-1)}{h-1}}{h} = \lim_{h\to 0} \frac{h}{h(h-1)} = \lim_{h\to 0} \frac{1}{h-1}\underline{-1}$$

(g)

$$f'(4) = \lim_{h\to 0} \frac{\frac{-3}{4+h} - \frac{-3}{4}}{h} = \lim_{h\to 0} \frac{\frac{-3\cdot 4+3(4+h)}{4(4+h)}}{h} = \lim_{h\to 0} \frac{3h}{4(4+h)h} = \lim_{h\to 0} \frac{3}{4(4+h)} = \underline{\frac{3}{16}}$$

(i)

$$f'(7) = \lim_{h\to 0} \frac{4-4}{h} = \lim_{h\to 0} \frac{0}{h} = \underline{0}$$